SMJK YU HUA, KAJANG
Trial Examination, 2008
Mathematics S/T
Paper 1

Form : 6AS1, 6AS2, 6AS3, 6AK1
(75 candidates)
Date : 22 Sept 2008
Time : (9.30-12.30) am

Prepared by : Rosmaya Mokhtar
Checked by : Chew A S
Verified by : Head of Department

Instructions to candidates :

Answer all questions. All necessary working should be shown clearly. Non-exact numerical answers may be given correct to three significant figures or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

1. Express the infinite recurring decimal $0.6\dot{1}2\dot{5}$ (= 0.6125125125…) as a fraction in its lowest terms.

[4 marks]

2. If $y = \dfrac{x}{x^3 - 1}$, show that $x^2 \dfrac{dy}{dx} = -y^2(2x^3 + 1)$

[4 marks]

3. If $\log_b\left(\dfrac{y}{b^2}\right) = \log_b 3 + \log_b 5 - \log_b (y + 2b)$, express y in terms of b.

[6 marks]

4. Simplify

(a) $\dfrac{3(\sqrt{5} + \sqrt{2})^2}{\sqrt{5} - \sqrt{2}}$ [3 marks]

(b) $\dfrac{3(1 - i)}{(2 + 3i)}$, where $i^2 = -1$ [3 marks]

5. A point moves such that the sum of the squares of its distances from three points (0,4), (0,–4) and (6,3) is 362. Find the equation of the locus of the point. Show that this locus is a circle passing through (8,9), and find its centre and radius.

[6 marks]

6. Find

(a) $\displaystyle\int \frac{x^2}{x^2 - 2x + 1}\,dx$ [3 marks]

(b) $\displaystyle\int x^3 e^{x^2}\,dx$ [4 marks]

7. Find the constants A, B, C and D such that

$$\frac{5x^3 + 31x^2 + 36x + 27}{x^2(x+3)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+3)} + \frac{D}{(x+3)^2}$$

[8 marks]

8. The function f is defined by

$$f(x) = \begin{cases} |4x - x^2| & , x \le 4 \\ x^2 - 5x + 6 & , x > 4 \end{cases}$$

(a) Find $\lim_{x \to 0^-} f(x)$, $\lim_{x \to 0^+} f(x)$, $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$ [4 marks]

(b) Determine whether f is continuous at $x = 0$ and $x = 4$. [4 marks]

9. The matrices X and Y are given by

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} , \quad Y = \begin{pmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

Show that $XY = kI$, where k is a constant and I is the 3×3 identity matrix. Hence, deduce the inverse of X. [5 marks]

Solve the system of linear equations.

$$x + 2y + 3z = 26$$
$$2x + 4y + 5z = 46$$
$$3x + 5y + 6z = 57$$

[5 marks]

10. The gradient of the tangent to a curve at any point (x,y) is given by

$$\frac{dy}{dx} = \frac{2(1 - x^{\frac{1}{3}})}{x^{\frac{1}{3}}}$$

where $x > 0$, If the curve passes through the point $(8, -4)$,

(a) find the equation of the curve [4 marks]
(b) sketch the curve [2 marks]
(c) calculate the area of the region bounded by the curve and the x-axis. [5 marks]

11. Using the substitution $y = x + \frac{1}{x}$, express $f(x) = x^3 - 2x^2 - x + 4 - \frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^3}$ as a polynomial in y. [3 marks]

Hence, find all the real roots of the equation $f(x) = 0$. [10 marks]

12. Find the coordinates of the stationary points on the curve $y = \frac{2x}{x^2 + 1}$ and determine their nature. [10 marks]

Sketch the curve. [4 marks]

Determine the number of real roots of the equation $2x = k(x^2 + 1)$, where $k \in R$, when k varies. [3 marks]

SMJK YU HUA, KAJANG

STPM Trial Examination 2008

Subject	: 954/2 Mathematics T Paper 2	Prepared by	: Chew AS
Form	: 6AS1, 6AS2, 6AS3	Checked by	: Lee SC
Date	: 19 September 2008 (Friday)	Verified by	: Head of Department
Time	: 0930-1230 (3 hours)		

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Instructions to Candidates

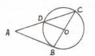
Answer all questions. All necessary working must be shown clearly. Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. Mathematical tables, a list of mathematical formulae and graph paper are provided.

1. Express $4\sin\theta - 3\cos\theta$ in the form $r\sin(\theta-\alpha)$, where $r>0$ and $0<\alpha<90°$. Hence, solve the equation.
$$4\sin\theta - 3\cos\theta = 3$$
for $0° < \theta < 360°$ [6]

2. Prove the following identities:
   (a) $2\cos\theta(\sin 3\theta - \sin\theta) = \sin 4\theta$
   (b) $4(\cos 4\theta + \cos 2\theta)\sin 3\theta \sin\theta = \cos 4\theta - \cos 8\theta$ [6]

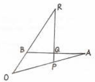
   By substituting θ with a suitable value in the above identities, prove that
   $$\sin 54° - \sin 18° = \frac{1}{2}, \text{ and}$$
   $$\sin 54° \sin 18° = \frac{1}{4}.$$ [4]

3.



In the above diagram, BC is a diameter of a circle centre O and D is a point on the circumference nearer to B than C. If CD is produced to A so that CD = DA, prove that
(a) the triangles CDB, ADB are congruent. [3]
(b) DO is parallel to AB. [3]
Given that BC = 10 cm and $\angle DCB = 35°$, calculate the area of the triangle ABC. [4]

4.



The diagram above shows non-collinear points O, A and B, with P on the line OA such that OP:PA = 2:1 and Q on the line AB such that AQ:QB = 2:3. The lines PQ and OB produced meet at the point R. If $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

(a) show that $\overrightarrow{PQ} = -\frac{1}{15}\mathbf{a} + \frac{2}{5}\mathbf{b}$ [5]

(b) find the position vector of R, relative to O, in terms of **b**. [5]

5. Two ships P and Q are 2 km apart. Ship P sails at a velocity of 3 m s^{-1} to the east and ship Q sails at a velocity of 4 m s^{-1} in the direction N60°E. Find

(a) the speed and direction of Q relative to P. [4]

(b) the time taken to the nearest minute for Q to be due east of P. [3]

6. By using the substitution $z = x + y$, find the solution to the differential equation

$\frac{dy}{dx} = \frac{x+y+3}{x+y-3}$ given that $y = 1$ when $x = 0$. [8]

7. John writes 6 letters, one each to P, Q, R, S, T and U. Each letter is placed in a separate envelope and sealed. He then addresses the envelopes, at random, one each to P, Q, R, S, T and U.

(a) Find the probability that the letter to P is in the correct envelope and the letter to Q is in an incorrect envelope. [2]

(b) Find the probability that the letter to P is in the correct envelope, given the letter to Q is in an incorrect envelope. [3]

(c) Find the probability that both of the letters to P and Q are in the incorrect envelopes. [3]

8. In a study of women above 50 years, it was found 2% of them suffer from osteoporosis.

(a) If a sample of 10 women is selected randomly, determine the probability that

(i) exactly two women [2]

(ii) at least two women suffer from osteoporosis. [3]

(b) A random sample of 200 women is obtained. Using a suitable approximation, determine the probability that at most two women suffer from osteoporosis. [3]

9. An unbiased cubical die has the number 1 on three faces, the number 2 on two faces and the number 3 on one face. The die is rolled twice and X is the total score. Find the probability distribution of X. [2]

Show that $E(X)=\frac{10}{3}$ and $Var(X)=\frac{10}{9}$. [5]

10. The marks in a certain examination were normally distributed with mean μ and standard deviation σ. 5% of the candidates had more than 85 marks and 10% had less than 25 marks. Find the values of μ and σ. [6]

11. The continuous random variable X has probability density function, f given by

$$f(x)=\begin{cases} c & ,0\le x<2 \\ c(3-x) & ,2<x\le 3 \\ 0 & ,otherwise \end{cases}$$, where c is a constant.

(a) Show that $c=\frac{2}{5}$. [3]

(b) Sketch the graph of f. [2]

(c) Find k such that $P(X\le k)=\frac{9}{10}$, giving your answer correct to 3 decimal places. [5]

12. The table shows the cumulative frequency distribution for the heights of 400 children in a school.

Height	<90	<100	<110	<120	<130	<140	<150	<160
Frequency	0	25	86	225	310	365	390	400

(a) Draw a cumulative frequency curve for the data given.
Using the curve, find the median height, the lower quartile and the upper quartile. [7]

(b) Calculate the mean height of the children. [3]